45.2A: Exponential Population Growth

When resources are unlimited, a population can experience exponential growth, where its size increases at a greater and greater rate.

Learning Objectives

• Describe exponential growth of a population size

Key Points

• To get an accurate growth rate of a population, the number that died in the time period (death rate) must be removed from the number born during the same time period (birth rate).

• When the birth rate and death rate are expressed in a per capita manner, they must be multiplied by the population to determine the number of births and deaths.

• Ecologists are usually interested in the changes in a population at either a particular point in time or over a small time interval.

• The intrinsic rate of increase is the difference between birth and death rates; it can be positive, indicating a growing population; negative, indicating a shrinking population; or zero, indicating no change in the population.

• Different species have a different intrinsic rate of increase which, when under ideal conditions, represents the biotic potential or maximal growth rate for a species.

Key Terms

• **fission**: the process by which a bacterium splits to form two daughter cells
In his theory of natural selection, Charles Darwin was greatly influenced by the English clergyman Thomas Malthus. Malthus published a book in 1798 stating that populations with unlimited natural resources grow very rapidly, after which population growth decreases as resources become depleted. This accelerating pattern of increasing population size is called exponential growth.

The best example of exponential growth is seen in bacteria. Bacteria are prokaryotes that reproduce by prokaryotic fission. This division takes about an hour for many bacterial species. If 1000 bacteria are placed in a large flask with an unlimited supply of nutrients (so the nutrients will not become depleted), after an hour there will be one round of division (with each organism dividing once), resulting in 2000 organisms. In another hour, each of the 2000 organisms will double, producing 4000; after the third hour, there should be 8000 bacteria in the flask; and so on. The important concept of exponential growth is that the population growth rate, the number of organisms added in each reproductive generation, is accelerating; that is, it is increasing at a greater and greater rate. After 1 day and 24 of these cycles, the population would have increased from 1000 to more than 16 billion. When the population size, N, is plotted over time, a J-shaped growth curve is produced.

The bacteria example is not representative of the real world where resources are limited. Furthermore, some bacteria will die during the experiment and, thus, not reproduce, lowering the growth rate. Therefore, when calculating the growth rate of a population, the death rate (D; the number organisms that die during a particular time interval) is subtracted from the birth rate (B; the number organisms that are born during that interval). This is shown in the following formula:

\[
\frac{\Delta N}{\Delta T} = B - D \Delta N / \Delta T = B - D
\]

where \( \Delta N \) = change in number, \( \Delta T \Delta T = \) change in time, \( BB = \) birth rate, and \( DD = \) death rate. The birth rate is usually expressed on a per capita (for each individual) basis. Thus, \( B \) (birth rate) = \( bN \) (the per capita birth rate “\( b \)” multiplied by the number of individuals “\( N \)”) and \( D \) (death rate) = \( dN \) (the per capita death rate “\( d \)” multiplied by the number of individuals “\( N \)”). Additionally, ecologists are interested in the population at a particular point in time: an infinitely small time interval. For this reason, the terminology of differential calculus is used to obtain the “instantaneous” growth rate, replacing the change in number and time with an instant-specific measurement of number and time.
\[ \frac{dN}{dT} = BN - (BD)N \]

Notice that the "d" associated with the first term refers to the derivative (as the term is used in calculus) and is different from the death rate, also called "d." The difference between birth and death rates is further simplified by substituting the term "r" (intrinsic rate of increase) for the relationship between birth and death rates:

\[ \frac{dN}{dT} = rN \]

The value "r" can be positive, meaning the population is increasing in size; negative, meaning the population is decreasing in size; or zero, where the population’s size is unchanging, a condition known as zero population growth. A further refinement of the formula recognizes that different species have inherent differences in their intrinsic rate of increase (often thought of as the potential for reproduction), even under ideal conditions. Obviously, a bacterium can reproduce more rapidly and have a higher intrinsic rate of growth than a human. The maximal growth rate for a species is its biotic potential, or \( r_{max} \), thus changing the equation to:

\[ \frac{dN}{dT} = r_{max}N \]